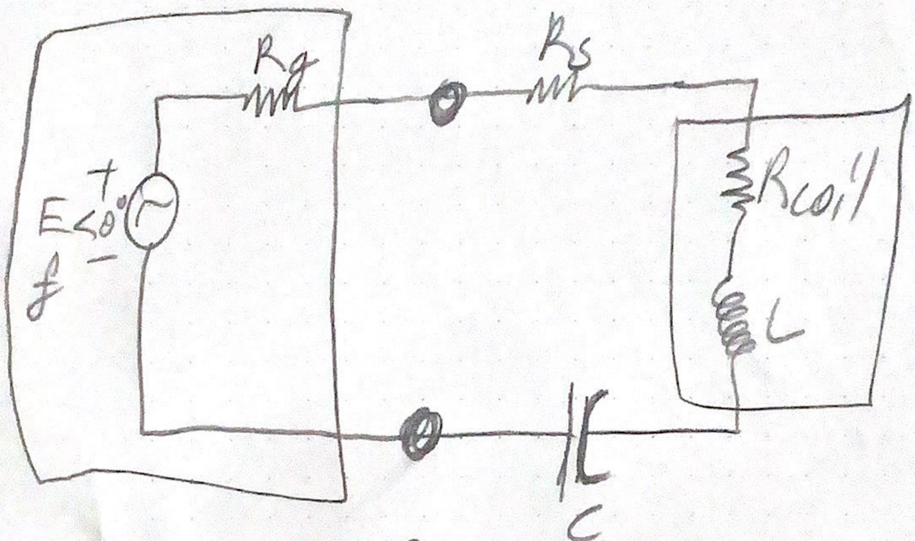
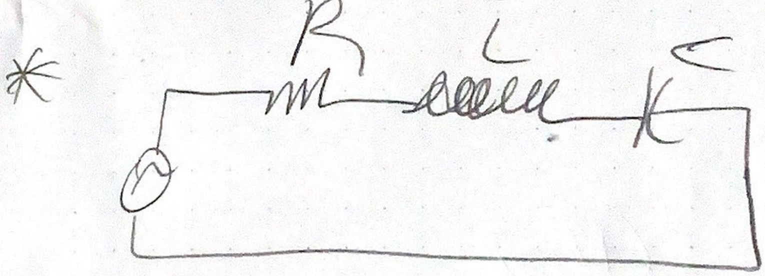
Resonance

*It is one of the most important circuit used in electronic.

1- Series Resonant Circuit

2- Parallel Resonant Circuit





$$R = R_g + R_s + R_{coil}$$

$$Z_T = R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

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* Z_T is purely resistive. Inductor and capacitor cancel each other.

$$\boxed{Z_T = R}$$

thus,

$$\underbrace{wL}_{X_L} = \frac{1}{\underbrace{wC}_{X_C}}$$

$$\bigstar \boxed{w_s = \frac{1}{\sqrt{LC}}} \text{ (rad/s)}$$

$$w = 2\pi f$$

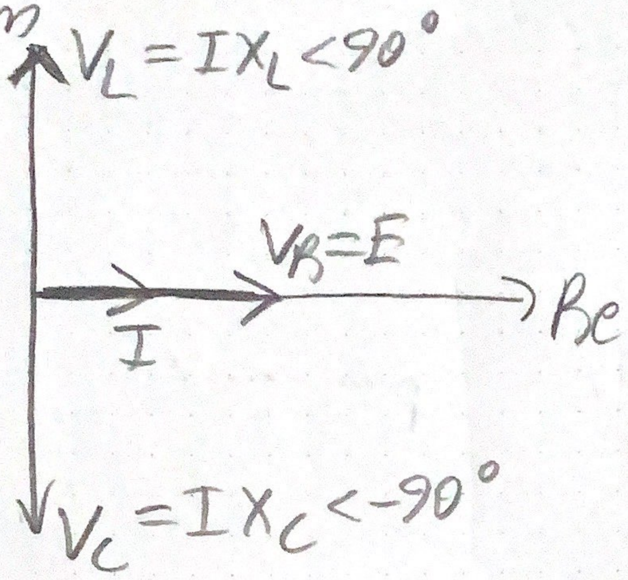
$$\frac{1}{\sqrt{LC}} = 2\pi f$$

$$\bigstar \boxed{f_s = \frac{1}{2\pi\sqrt{LC}}} \text{ (Hz)}$$

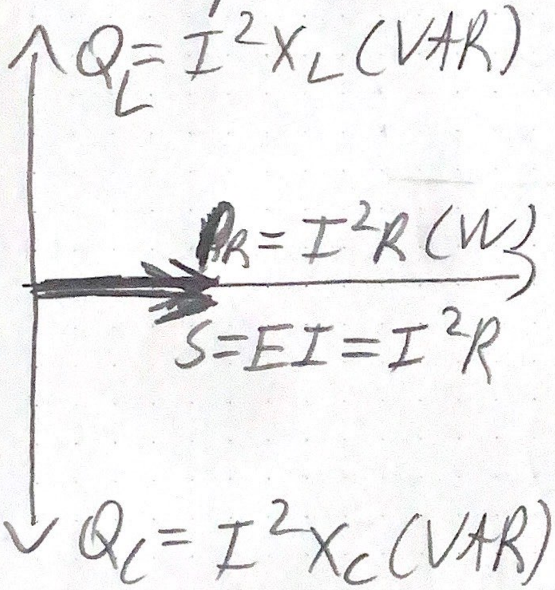
* At resonance ~~total~~ current in the circuit phasor

$$I = \frac{E}{Z_T} = \frac{E\angle 0^\circ}{R\angle 0^\circ} = \frac{E}{R}\angle 0^\circ$$

* Im



* Let's determine average power and reactive powers



* Quality Factor;

$$\underset{\text{Quality factor}}{\underline{Q}} = \frac{\text{reactive power}}{\text{active power}} \Rightarrow Q_s = \frac{I^2 \overset{wL}{\widetilde{X_L}}}{I^2 R} = \boxed{\frac{wL}{R}}$$

→ important

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*If inductor has coil

$$\boxed{Q_{coil} = \frac{X_L}{R_{coil}}}$$

$$\boxed{* \; Q_S = \frac{\overbrace{IX_L}^{V_L}}{\underbrace{IR}_{E}} = \frac{V_L}{E}}$$

$$\boxed{V_C = V_L = Q_S E}$$! at "resonance"

Impedance Of a Series Resonant Circuit

$$* \; Z_T = R + jwL - \frac{j}{wC}$$

$$= R + j\underbrace{\left(wL - \frac{1}{wC}\right)}_{\left(\frac{w^2LC - 1}{wC}\right)}$$

$$\boxed{\begin{aligned} Z_T &= \sqrt{R^2 + \left(\frac{w^2LC - 1}{wC}\right)^2} \\ \theta &= \tan^{-1}\left(\frac{w^2LC-1}{wRC}\right) \end{aligned}}$$

1- When $w = w_s$

$Z_T = 0$

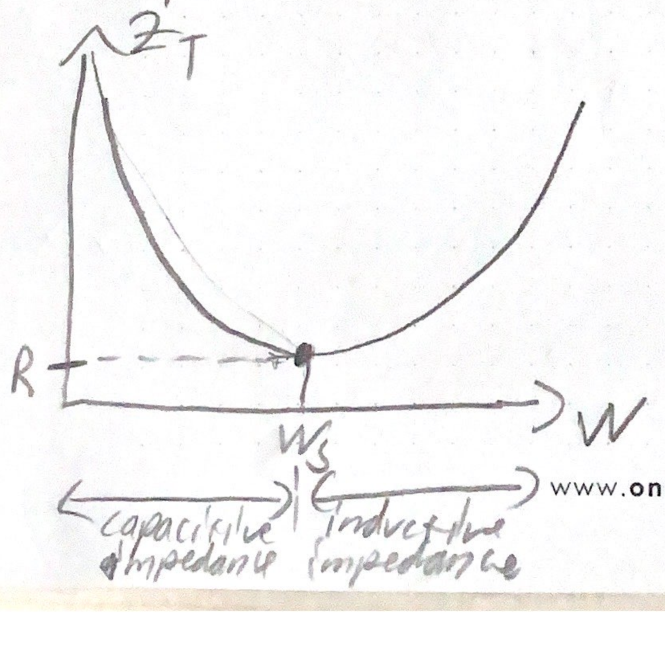
$\theta = \tan^{-1} 0 = 0$

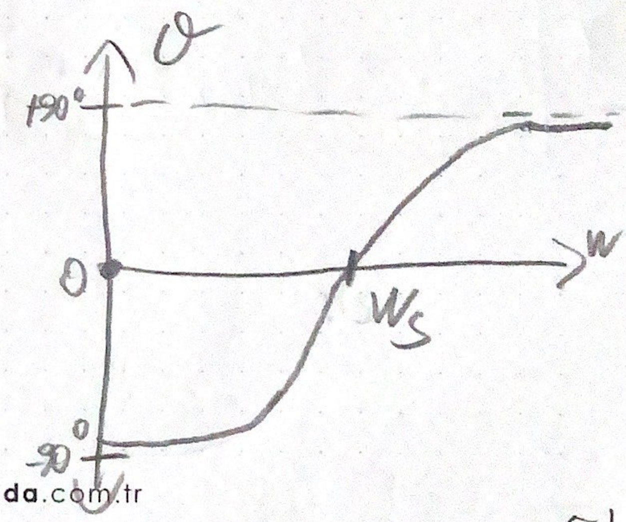
2- When $w < w_s$

capacitor behaves like open circuit in DC. To sum up, impedance must appear CAPACITIVE in this region

3- When $w > w_s$

Impedance must be inductive.





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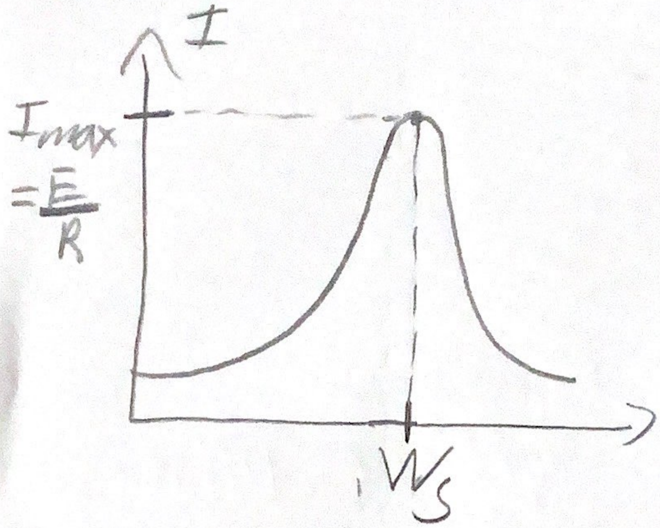
Power, Bandwidth and Selectivity of a Series Resonant Circuit.

$$I_{max} = \frac{E}{R} \Big\} \text{ at resonance}$$

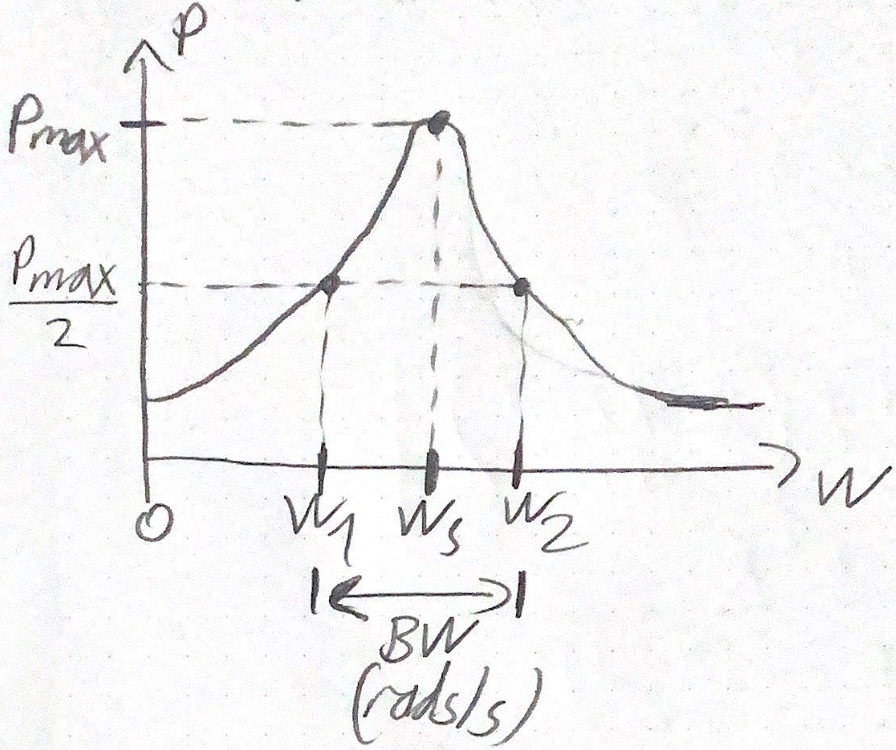


* total power at any frequency

$$\boxed{P = I^2 \cdot R}$$

* $\boxed{P_{max} = I^2_{max} R = \frac{E^2}{R}}$ at resonance

Selectivity Curve



-if BW very narrow, circuit have high selectivity. Otherwise, if it's large have low selectivity power at any frequency,

$$P = I^2 R$$

$$P = \left(\frac{E}{Z_T}\right)^2 \cdot R$$

$$\left[R + j\left(\frac{w^2LC - 1}{wC}\right)\right]^2$$

$$P = \frac{E^2 R}{R^2 + \left(\frac{w^2LC-1}{wC}\right)^2}$$

* half power frequencies, the power must be

$$\boxed{\begin{gathered} P_{hpf} = \frac{E^2}{2R} \\ I_{hpf} = \sqrt{\frac{P_{hpf}}{R}} = \underbrace{\sqrt{\frac{E^2}{2R^2}}}_{I_{max} = \frac{E}{R}} = \sqrt{\frac{I_{max}^2}{2}} \\ I_{hpf} = \frac{I_{max}}{\sqrt{2}} \end{gathered}}$$

* $\boxed{w^2 LC - 1 = wRC \text{ (at half power)}}$

$$\boxed{\begin{aligned} w_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ w_2 &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{aligned}}$$

önemli

* $BW = w_2 - w_1$

$\boxed{BW = \frac{R}{L}}$ (rad/s)

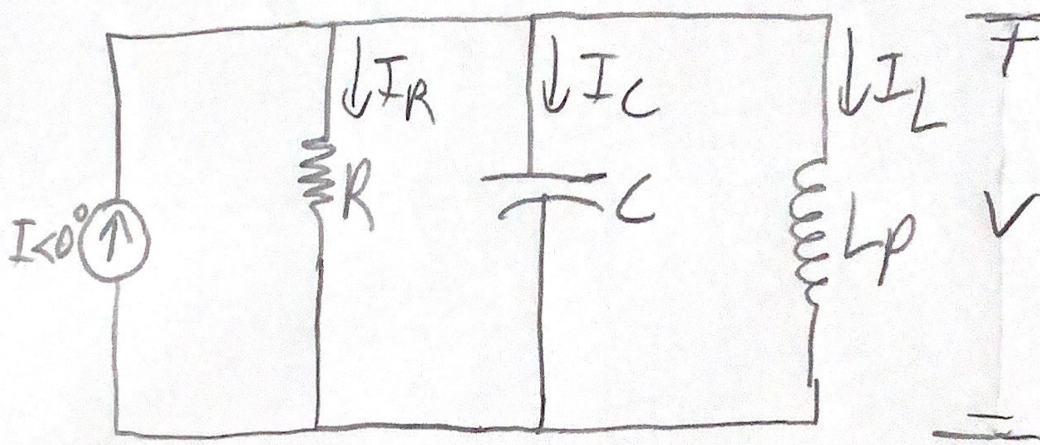
$$BW = \frac{w_s R}{w_s L} = \boxed{\frac{w_s}{Q_s}} \text{ (rad/s)}$$

X_L

$$\boxed{* BW = \frac{f_s}{Q_s}}\ (Hz)$$

Parallel Resonance

best analyzed using a constant current source.

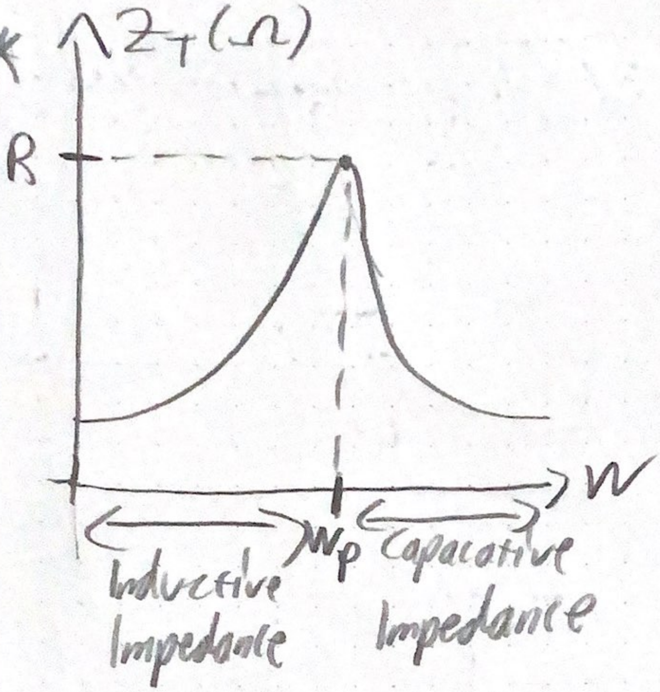


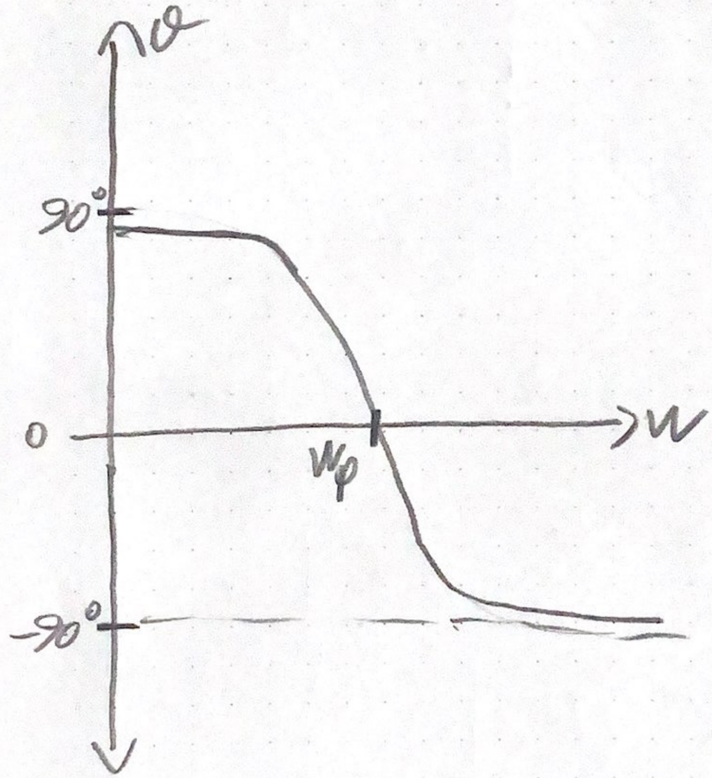
$$\star\ W_p = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{R_{coil}^2 \cdot C}{L}}$$

önemli:

$$\star\ W_p \cong \frac{1}{\sqrt{LC}}, \quad \text{for } L/C \geq 100R_{coil}$$

*


$Z_T(\Omega)$
R
w
w_p
Inductive Impedance
Capacative Impedance


θ
90°
0
w_p
w
-90°

$$Q_p = \frac{\text{reactive power}}{\text{average power}}$$

$$= \frac{V^2/X_L}{V^2/R}$$

$$I_R = \frac{V}{R} = I$$

$$I_L = \frac{V}{X_L \angle 90^\circ}$$

$$= \frac{V}{R/Q_p} \angle -90^\circ$$

$$= Q_p I \angle -90^\circ$$

$$I_C = \frac{V}{X_C \angle -90^\circ}$$

$$= \frac{V}{R/Q_p} \angle 90^\circ$$

$$= Q_p I \angle 90^\circ$$

★ Half power frequencies,

★ $$w_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \ (rad/s)$$

★ $$w_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

★ $$BW = w_1 - w_2 = \frac{1}{RC} \ (rad/s)$$

★ $$BW = \frac{w_p}{Q_p} \ (rad/s)$$

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